Pluralities of individual concepts are needed*

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Abstract

Any grammar capable of handling plurality and intensionality must integrate the two in some way. With respect to the denotation of plural noun phrases, two options arise, depending on whether intensionalization precedes pluralization or vice versa: they can either denote *intensional plurals* (i.e., functions from indices to sets of individuals) or *pluralities of concepts* (i.e., sets of individual concepts). While the former is standard, we show that pluralities of concepts are needed once we consider sentences with predicates that require their arguments to be both plural and intensional.

1 Overview

Plurality and intensionality each call for a richer notion of individuals. In (1a), the subject denotes a plural individual, or *plurality*, i.e., a set of ordinary individuals. And in (2a), it denotes an *individual concept*, i.e., a function from indices of evaluation to individuals.

(1) a. Ann and Beth met.
b. $[\![\text{Ann and Beth}]\!] = \{\mathsf{ann}, \mathsf{beth}\}$ $\{\mathsf{e}\}$

(2) a. The mayor changed.
b. $[\![\text{the mayor}]\!] = \lambda i.\iota x[\mathsf{mayor}(i)(x)]$ $\mathsf{s} \to \mathsf{e}$

This paper investigates how these two enrichments should be combined. In particular, we compare two analyses of plural noun phrases such as *the governor and the mayor*, shown in (3). On one analysis, we intensionalize a plural individual to yield an *intensional plural*—i.e., a function that maps indices to pluralities. On the other, we pluralize an individual concept to yield a *plurality of concepts*—i.e., a collection of functions from indices to individuals.

(3) a. $\lambda i.\{[\![\text{the governor}]\!]^i, [\![\text{the mayor}]\!]^i\}$ *an intensional plural*
b. $\{(\lambda i.[\![\text{the governor}]\!]^i), (\lambda i.[\![\text{the mayor}]\!]^i)\}$ *a plurality of concepts*

*To be added.

The objects in (3a) and (3b) are far from being notational variants: pluralities of concepts encode strictly more information than intensional plurals. Suppose, for example, Ann is the mayor and Beth is the governor at some index i, but the two of them swap positions at i'. The intensional plural in (3a) can't distinguish i from i', since it yields the same plurality at both indices. However, we can always keep track of who is the mayor and who is the governor with (3b)— the two concepts are still independently accessible. A grammar that generates pluralities of concepts is therefore strictly more expressive than one that doesn't. We call the simpler grammar Theory A, and the more powerful one Theory B. The question, then, is whether natural language data calls for the richer notion of plurality afforded by Theory B or whether Theory A is sufficient.

Three case studies will show that Theory B is in fact necessary: intensional plurals prove to be too coarse. This is an important result, since Theory A is the most widely adopted way of combining plurality and intensionality— a consequence of the view that plurality pertains to the domains of individuals (e.g., Link 1983). The idea that individual concepts themselves can be pluralized is not novel, however. Pluralities of concepts have been explicitly argued for in Krifka (2009) and Haslinger and Schmitt (2021), and the general theory of plurality developed by Schmitt (2019) subsumes them. What is novel about our paper is the nature of the argumentation: we combine independently motivated arguments for plural individuals and for individual concepts, and show that, taken together, only pluralities of concepts adequately account for the relevant data. Thus, the necessity of pluralities of concepts follows directly from accepting these standard premises.

The remainder of the paper is organized as follows: §2 presents Theory A and Theory B; §3, the core of the paper, presents the evidence for Theory B; §4 addresses the question of whether there is room for intensional plurals in a grammar that already generates pluralities of concepts; §5 concludes and discusses some open questions.

2 Combining plurality and intensionality

This section introduces the ingredients needed for our discussion. First, we present the machinery independently required to account for plurality and intensionality. Then, we show how these can be combined in two different ways, corresponding to two distinct grammars.

2.1 Generating pluralities

We adopt a standard set-theoretic treatment of pluralities (Bennett 1974; Winter 2001), illustrated in (4). We use $\{\alpha\}$ as a notational shorthand for $\alpha \to \mathsf{t}$, i.e., the type of any plurality composed of objects of type α.

(4) $[\![\text{Ann and Beth}]\!] = \{\mathsf{ann}, \mathsf{beth}\}$ $\{\mathsf{e}\}$

Collective predicates denote predicates of pluralities, while distributive predicates denote predicates of ordinary individuals, as shown in (5). Plural noun phrases

can compose with a distributive predicate via a covert distributivity operator Δ, defined in (6).

(5) a. $[\![\text{meet}]\!] = \lambda X : \#(X) > 1.\text{meet}(X)$ $\quad \{e\} \to t$

b. $[\![\text{tall}]\!] = \lambda x.\text{tall}(x)$ $\quad e \to t$

(6) $[\![\Delta]\!] = \lambda P.\lambda X : X \neq \varnothing.\, \forall x \in X.\, P(x)$ $\quad (e \to t) \to \{e\} \to t$

We will focus on non-quantificational plural noun phrases, that is, coordinations and plural definites. To account for the former, we analyze *and* as set union:

(7) $[\![\text{and}]\!] = \lambda X.\lambda Y.X \cup Y$ $\quad \{e\} \to \{e\} \to \{e\}$

This analysis doesn't allow us to combine *and* directly with singular DPs, so we use Partee's (1986) type-shifter ID, defined in (8), to shift singularities into singleton pluralities. The structure underlying *Ann and Beth* would thus be as in (9).

(8) $[\![\text{ID}]\!] = \lambda x.\, \{x\}$ $\quad e \to \{e\}$

(9) [ID Ann] and [ID Beth]

To interpret plural definites, we first derive the meaning of plural nouns by applying Δ to the singular noun:

(10) $[\![\text{mayors}]\!] = [\![\Delta]\!]([\![\text{mayor}]\!])$
$= \lambda X : X \neq \varnothing.\forall x \in X.\, [\![\text{mayor}]\!](x)$

We then adopt the maximality-based analysis of the definite determiner in **sharvy1980more**. However, because we take singularities and pluralities to have different types, *the* is assigned the type-flexible denotation in (11), where $\subseteq_\alpha$ is the subset relation when α is of a boolean type but the identity relation otherwise. In (12) we illustrate how singular and plural definites are interpreted.

(11) $[\![\text{the}_\alpha]\!] = \lambda f.\iota x.f(x) \wedge \forall y.f(y) \to y \subseteq_\alpha x$ $\quad (\alpha \to t) \to \alpha$

(12) a. $[\![\text{the}_e \text{ mayor}]\!] = \iota x.\, \text{mayor}(x) \wedge \forall y \in \text{mayor}.\, y = x$

b. $[\![\text{the}_{\{e\}} \text{ mayors}]\!] = \iota X.\, (\forall x \in X.\text{mayor}(x)) \wedge \forall Y \subseteq \text{mayor}.\, Y \subseteq X$
$= \{x \mid \text{mayor}(x)\}$

2.2 Generating individual concepts

As reported in Montague 1973, Barbara Partee observed that certain predicates cannot be treated as predicates of ordinary individuals. This is illustrated by the following invalid argument, known as the *temperature paradox* or *Partee's paradox*:

(13) The temperature rises.
The temperature is ninety.
#. Ninety is rising.

If *rises* were a predicate of individuals, substitution of co-extensional descriptions should preserve truth. Montague (1973) thus proposed that predicates like *rise* compose not with ordinary individuals but with *individual concepts* (ICs), i.e., functions from indices of evaluation to individuals.[1]

We assume here that *rise* composes with its subject via a silent operator ^, which makes intensions available for composition (Keshet 2010):

(14) $[\![\hat{} \, \phi]\!]^i = \lambda j.[\![\phi]\!]^j$

(15) $[\![\hat{}[\text{the temperature}] \text{ rises}]\!]^t = [\![\text{rises}]\!]^t(\lambda t'.[\![\text{the temperature}]\!]^t)$

Just as NPs can denote predicates of pluralities, NPs can also denote predicates of ICs. This is motivated, for example, by the fact that relative clauses with intensional VPs can restrict NPs (Schwager 2007, Frana 2017):

(16) The temperature [that is rising now] wasn't rising yesterday.

We can maintain the intersective treatment of relative clauses if we assume that the operator IC, defined in (17), can shift NPs into predicates of ICs (Nathan 2006, Grosu and Krifka 2007):

(17) $[\![\text{IC}]\!]^t = \lambda N.\lambda u.\, \forall i \in \mathsf{dom}(u).\, N(i)(u(i))$

(18) $[\![\text{price that is rising}]\!]^t$
$= \lambda u.\, [\![\text{IC } \hat{}[\text{price}]]\!]^t(u) \wedge [\![\text{rising}]\!]^t(u)$
$= \lambda u.\, (\forall i.\, [\![\text{price}]\!]^i(u(i))) \wedge [\![\text{rising}]\!]^t(u)$

2.3 Intensional plurals or pluralities of concepts?

We now address the question of how the enrichments introduced in the previous two subsections are to be combined. As things stand, the options are limited — because *and* can only create plural individuals, the only intensional plural entities we can generate are *intensional plurals*:

(19) *Intensional plurals (Theory A)*

a. [^ [[ID the governor] and [ID the mayor]]]

b. $[\![(19a)]\!]^w = \lambda i.\, \{ \mathsf{the.gov}(i), \mathsf{the.may}(i) \}$ $\qquad \mathsf{s} \rightarrow \{\mathsf{e}\}$

In (19a), pluralization precedes intensionalization. We call a grammar that only permits this ordering of operations Theory A.

An alternative arises if intensionalization applies before pluralization, allowing concepts to be combined into pluralities. This will only be possible, however, if we allow the operators introduced in §2.1 to create pluralities of different types. We thus assign them type-flexible meanings:

[1] For maximal generality, indices of evaluation should be taken to be world-time pairs. However, to simplify our presentation, we'll consider them to be intervals of times or worlds depending on the kind of intensionality under discussion.

(20) a. $[\![\text{and}_\alpha]\!] = \lambda X.\lambda Y.\, X \cup Y$ $\{\alpha\} \to \{\alpha\} \to \{\alpha\}$

b. $[\![\text{ID}_\alpha]\!] = \lambda x.\, \{x\}$ $\alpha \to \{\alpha\}$

We can now have a structure that yields a *plurality of concepts*, as shown in (21). We will refer to this grammar as Theory B.

(21) *Pluralities of concepts (Theory B)*

a. [ID_{se} [^ the governor]] $\text{and}_{\{\text{se}\}}$ [ID_{se} [^ the mayor]]

b. $[\![(21a)]\!]^w = \{\, \lambda i.\, \text{the.gov}(i), \lambda i.\, \text{the.may}(i) \,\}$ $\{\text{s} \to \text{e}\}$

As already mentioned in the introduction, Theory B is strictly more expressive than Theory A. One way to see this is via the natural mapping from pluralities of concepts to intensional plurals. Given a plurality of concepts U, we can define a function **flat** that flattens U into an intensional plural as in (22). We show in (23) that we can use **flat** to go from (21b) to (19a).

(22) $\textbf{flat}(U) = \lambda i.\, \{\, u(i) \mid u \in U \,\}$ $\{\text{s} \to \text{e}\} \to \text{s} \to \{\text{e}\}$

(23) $\textbf{flat}(\{\, \lambda i.\, \text{the.gov}(i), \lambda i.\, \text{the.may}(i) \,\}) = \lambda i.\, \{\, \text{the.gov}(i), \text{the.may}(i) \,\}$

Crucially, **flat** is many-to-one, i.e., not injective, and hence has no inverse. For example, **flat** maps the two different pluralities of concepts in (24) to the same intensional plural in (25). Therefore, pluralities of concepts encode more information.

(24) a. $\left\{ \begin{bmatrix} \text{w} & \mapsto & \text{beth} \\ \text{v} & \mapsto & \text{beth} \end{bmatrix}, \begin{bmatrix} \text{w} & \mapsto & \text{ann} \\ \text{v} & \mapsto & \text{ann} \end{bmatrix} \right\}$

b. $\left\{ \begin{bmatrix} \text{w} & \mapsto & \text{ann} \\ \text{v} & \mapsto & \text{beth} \end{bmatrix}, \begin{bmatrix} \text{w} & \mapsto & \text{beth} \\ \text{v} & \mapsto & \text{ann} \end{bmatrix} \right\}$

(25) $\begin{bmatrix} \text{w} \mapsto \{\text{ann}, \text{beth}\} \\ \text{v} \mapsto \{\text{ann}, \text{beth}\} \end{bmatrix}$

Greater expressive power by itself does not entail empirical adequacy, however — the additional information afforded by Theory B is only justified if it is required by the data. The central question left to be answered is whether this additional expressive power is empirically motivated.

3 Comparing the two approaches

In this section, we provide evidence that Theory A is insufficient to account for the interaction between plurality and intensionality. We do so through three case studies: (i) collective intensional VPs (§3.1), (ii) the interaction of distributivity and intensional VPs (§3.2), and (iii) plural NPs (§3.3).

3.1 The argument from collective intensional VPs

In §2.1, we saw two kinds of VPs that don't denote predicates of ordinary individuals: collective VPs like *meet* and intensional VPs like *change*. We now consider predicates that combine both properties — what we call *collective intensional* VPs. These predicates can be diagnosed by two properties: they require a plural argument, and they give rise to failures of substitution. We will argue that some of these predicates can only be accounted for under Theory B, but first we will present some that are compatible with both approaches.

A good candidate for a VP in this class is *be the same person*:

(26) a. The governor and the mayor are the same person.
≈ *'the governor' refers to the same individual 'the mayor' does*

b. *The governor is the same person.

The ungrammaticality of (26b) clearly shows that *be the same person* is collective, but it may also suggest that we are dealing with an intensional predicate. After all, if the governor and the mayor are the same person, then *the governor and the mayor* denotes the very same individual as *the governor*. Thus, the fact that substitution is not truth preserving suggests that we might not be dealing with an extensional VP.

This predicate has received some attention in the literature, with at least two works being relevant to us. Focusing on embedded occurrences of *be the same person*, Zhang (2016) proposes an analysis that appeals to an enriched notion of pluralities (i.e., lists rather than sets), while Percus (2019) suggests that it denotes a predicate of pluralities of ICs, exactly in line with the more expressive Theory B. Although these works are not primarily concerned with the theory of plurality, both effectively assume that Theory A lacks the resources to account for such predicates. We now assess whether this is indeed the case or whether other kinds of VPs can provide more compelling evidence for Theory B.

Under Theory B, the analysis is straightforward: *be the same person* is true of any plurality of concepts whose parts point to the same individual (Percus 2019).

(27) $[\![\text{ be the same person }]\!]^w = \lambda U.\, \forall u, v \in U.\, u(w) = v(w)$ $\quad \{s \to e\} \to t$

This cannot be done in Theory A, because the concepts denoted by *the governor* and by *the mayor* simply cannot be retrieved from the intensional plural denoted by *the governor and the mayor*, as discussed in §2.3. However, it is not impossible to provide a purely extensional analysis of this predicate:

(28) $[\![\text{ be the same person }]\!]^w = \lambda X.\, |X| = 1$ $\quad \{e\} \to t$

This analysis assigns the correct truth conditions to (26a): *the governor and the mayor* denotes a singleton plurality just in case the governor and the mayor are the same person. This could even account for (26b): one could argue that its unacceptability is due to the fact that the sentence is a logical tautology whenever *be the same person*

composes with a plurality that is necessarily a singleton.[2] Therefore, *be the same person* does not by itself rule out Theory A.

There are also some intensional collective VPs that Theory A is well-suited to account for. Consider the predicate *be converging*: (29) shows that the VP is collective, and (30) that it is intensional.

(29) a. The temperatures of the kitchen and of the room are converging.
b. *The temperature of the kitchen is converging.

(30) The temperatures of the kitchen and of the room are converging.
The temperature of the kitchen is 60.
The temperature of the room is 55.
#. 60 and 55 are converging.

Theory A can account for this predicate because its truth conditions can be stated solely in terms of the range of values within the plurality at each index — we don't need to track which value is associated with which temperature. Setting aside the complexities of the progressive aspect, we can treat *be converging* as the predicate of intensional plurals in (31): it's true of any $\mathcal{X}$ at i as long as the difference between the highest and the lowest values in $\mathcal{X}(t_{\text{beg}})$ is greater than the difference between the highest and the lowest values in $\mathcal{X}(t_{\text{end}})$.

(31) $[\![\text{ be converging }]\!]^t \qquad (s \to \{e\}) \to t)$
$= \lambda\mathcal{X}.\ (\max(\mathcal{X}(t_{\text{beg}})) - \min(\mathcal{X}(t_{\text{beg}})) > (\max(\mathcal{X}(t_{\text{end}})) - \min(\mathcal{X}(t_{\text{end}}))$

A decisive argument for Theory B, however, can be made with predicates such as *crossed* or *reversed*. Intuitively, sentence (32) is true only if the price that was higher at an earlier time is the price that is lower at a later time. Sentence (33) is true only if the price associated with oat milk at an earlier time is the price associated with almond milk at a later time, and vice versa. These predicates differ from *converge* in that their truth conditions crucially require us to keep track of which value is associated with which price.

(32) The prices of oat milk and almond milk crossed.
*cf. *The price of oat milk crossed.*

(33) The prices of oat milk and almond milk are now reversed.
*cf. *The price of oat milk is now reversed.*

We can show that Theory A cannot account for these sentences in the following way. Consider the two prices scenarios in Table 1: both (32) and (33) are true in Scenario I and false in Scenario II. We treat each scenario as a distinct model with two intervals, April and May. Under Theory B, the DP *the prices of oat milk and almond milk* would denote different pluralities of concepts in each model:

[2] A particular challenge for this idea is that the predicate *be one person* does not lead to the same kind of unacceptability — *Mary is one person* sounds redundant, but not ungrammatical. In fact, it can even be used in particular contexts with focus on the numeral, e.g. *Mary is (just) ONE person*.

	Scenario I (crossing)		Scenario II (no crossing)	
	April	May	April	May
Oat milk	10	9	10	10
Almond milk	9	10	9	9

Table 1: Two price scenarios

(34) $\{\lambda i.\text{oat.price}(i), \lambda i.\text{alm.price}(i)\} =$

a. Scenario I: $\{[\text{april} \mapsto 10, \text{may} \mapsto 9], [\text{april} \mapsto 9, \text{may} \mapsto 10]\}$

b. Scenario II: $\{[\text{april} \mapsto 10, \text{may} \mapsto 10], [\text{april} \mapsto 9, \text{may} \mapsto 9]\}$

However, under Theory A the DP denotes the same intensional plural in both models:

(35) $\lambda i.\, \{\text{oat.price}(i), \text{alm.price}(i)\} =$

a. Scenario I: $[\text{april} \mapsto \{9, 10\}, \text{may} \mapsto \{9, 10\}]$

b. Scenario II: $[\text{april} \mapsto \{9, 10\}, \text{may} \mapsto \{9, 10\}]$

Since the denotation of the subject is identical across both scenarios under Theory A, any predicate meaning — regardless of its definition — will assign the same truth value in both cases. We thus conclude that no account for either *crossed* or *reversed* is possible within Theory A.

3.2 The argument from distributivity

Our second argument for Theory B comes from distributive inferences with intensional VPs. The contrast between (36) and (37) shows distributivity over ICs is possible: from *the governor and the mayor changed* we can infer that *change* is true of the intension of *the governor* but not of its extension.

(36) The governor and the mayor changed last year.
∴ The governor changed last year.

(37) The governor and the mayor changed last year.
Ann is the governor.
⊮ Ann changed last year.

Under Theory B, distributivity over ICs can be handled in essentially the same way as distributivity over ordinary individuals. All we need is the type-flexible entry for Δ in (38), which can apply to predicates of individuals as well as predicates of ICs. With this entry, the premise of (36) receives the straightforward analysis in (39).

(38) $[\![\, \Delta_\alpha \,]\!]^t = \lambda X.\lambda P.\, \forall x \in X.\, P(x)$ $\qquad (\alpha \to \mathsf{t}) \to \{\alpha\} \to \mathsf{t}$

	Scenario I (swapping)		Scenario II (no swapping)	
	2025	2026	2025	2026
the mayor	Ann	Beth	Ann	Ann
the governor	Beth	Ann	Beth	Beth

Table 2: Two election scenarios

(39) $[\![\text{the governor and the mayor changed}]\!]^t$
$= [\![\Delta_{se}]\!]^t([\![\text{change}]\!]^i)(\{\lambda i.\text{the.gov}(i), \lambda i.\ \text{the.may}(i)\})$
$= [\![\text{change}]\!]^t(\lambda i.\text{the.gov}(i)) \wedge [\![\text{change}]\!]^t(\lambda i.\text{the.may}(i))$

Theory A, however, cannot derive these entailments. The ordinary distributivity operator Δ_e quantifies over individuals, and it could only validate (36) if it also validated (37).

A natural response would be to enrich Theory A with a distributivity operator Δ' that can combine with *change* and yield a predicate of intensional plurals:

(40) $[\![\Delta']\!]([\![\text{change}]\!]^i) :: (s \to \{e\}) \to t$

The problem is that, to yield the desired results, any such Δ' would have to appeal to a putative function f that maps intensional plurals to the set of ICs that are its "parts":

(41) $f(\lambda i.\ \{\text{the.gov}(i), \text{the.may}(i)\}) = \{\lambda i.\ \text{the.gov}(i), \lambda i.\ \text{the.may}(i)\}$

But this f would amount to the inverse of the function **flat** (§2.3), which doesn't exist. Again, the issue is that a single intensional plural can correspond to multiple distinct pluralities of concepts, so there is no principled way to determine which ICs are its "parts."

We can make the same point more concretely via the scenarios in Table 2. In Scenario I, the governor and the mayor switch positions between 2025 and 2026; in Scenario II, neither role changes. Under its distributive reading, the sentence *the governor and the mayor changed* is therefore true in Scenario I but false in Scenario II. In Theory B, the subject *the governor and the mayor* denotes a different plurality of concepts in each scenario (assuming, again, that each scenario corresponds to a different model):

(42) $\{\lambda i.\text{the.gov}(i), \lambda i.\text{the.may}(i)\} =$

a. Scenario I: $\{[25 \mapsto \text{ann}, 26 \mapsto \text{beth}], [25 \mapsto \text{beth}, 26 \mapsto \text{ann}]\}$
b. Scenario II: $\{[25 \mapsto \text{ann}, 26 \mapsto \text{ann}], [25 \mapsto \text{beth}, 26 \mapsto \text{beth}]\}$

By contrast, under Theory A, the coordinated DP has the same denotation in both scenarios:

(43) $\lambda i.\ \{\text{the.gov}(i), \text{the.may}(i)\} =$

a. Scenario I: $[25 \mapsto \{\text{beth}, \text{ann}\}, 26 \mapsto \{\text{beth}, \text{ann}\}]$
b. Scenario II: $[25 \mapsto \{\text{beth}, \text{ann}\}, 26 \mapsto \{\text{beth}, \text{ann}\}]$

Since *the governor and the mayor* receives the same denotation in Scenarios I and II under Theory A, (44) must yield the same truth value in both models, regardless of the lexical entry assigned to Δ'.

(44) $[\![\Delta']\!]^t([\![\text{change}]\!]^t)([\![\text{the governor and the mayor}]\!]^t)$ where $t = 2025 \cup 2026$

3.3 The argument from plural NPs

Our final argument concerns plural NPs: we discuss three constructions in which a plural NP must be interpreted as a predicate of pluralities of concepts. The discussion in this subsection has some precedents in, among others, Gupta 1980, Condoravdi, Crouch, and Berg 2001 and, especially, Krifka 2009.

The first piece of evidence builds on the discussion in §3.1. In that section we established that *crossed* is a predicate of pluralities of concepts that cannot be reduced to a predicate of intensional plurals. Now, example (45) shows that a relative clause whose main predicate is *crossed* can modify the noun *prices*, which shows that this noun can be interpreted as a predicate of the same type, as in (46).

(45) The prices that crossed are now the same.

(46) $[\![\text{prices that crossed}]\!]^t = \lambda U.\ [\![\text{prices}]\!]^t(U) \wedge [\![\text{crossed}]\!]^t(U)$

Under Theory B, the analysis is straightforward. We assume that the singular noun *price* is shifted into a predicate of concepts via IC (§2.2), and then pluralized via Δ_{se}, yielding the predicate of pluralities of concepts in (47). Since Theory A cannot account for predicates such as *crossed*, it also fails here.

(47) $[\![\text{prices}]\!]^t = [\![\Delta_{se}]\!]^t([\![\text{IC}]\!]^t([\![\wedge \text{price}]\!]^t))$
$= \lambda U.\ \forall u \in U.\ \forall i \in \text{dom}(u).\ [\![\text{price}]\!]^i(u(i))$

There are other cases in which plural morphology seems to track concepts rather than individuals, such as:

(48) *Context*: The three of us have a class project that requires us to conduct three interviews. We divide up the work so that each of us interviews one person independently. When we meet to compare our data, we discover that…

The people we interviewed are all the same person.

Notably, the sentence above contains the plural noun *people* even though it entails that we only interviewed a single person. On the intended reading, the subject of (48) seems to be interpreted as the plurality containing ICs of the form *the person* α *interviewed*, where α is one of the interviewers. In addition, note that floated *all* is licensed, which can only be the case if it is associated with a DP whose cardinality is greater than two. Theory B can thus account for data like this.

Under Theory A, on the other hand, the subject of (48) denotes the following intensional plural: the function returning the set of individuals we interviewed at each index of evaluation. But this creates a dilemma. If the sentence is true, that set is a singleton, which makes the plural morphology on *people* and the licensing of floated *all* mysterious. If instead the concept returns a non-singleton plurality, the licensing of plural morphology and *all* is accounted for, but the sentence comes out as false. We thus conclude that the truth of (48) is a strong argument against Theory A.

The same point extends to numerals. For instance, (49) could also have been uttered in the scenario in (48):

(49) The three people we interviewed are all the same person.

Just like plural morphology, the example above shows that numerals can be used to count concepts instead of individuals. This is not a novel claim, however: for example, it is also found in Gupta 1980, Condoravdi, Crouch, and Berg 2001, Krifka 2009. We now briefly discuss some of the data discussed in these works and compare them to our own.

Gupta (1980) discusses data such as (50a), which can be true even if fewer than 10 million different people were transported: rather than 10 million individuals, we seem to be counting 10 million "time-slices" of individuals traveling inside a plane. Krifka (1990) reanalyzed these data as involving the counting of *events* rather than individual concepts: (50a) involves 10 million events of a passenger being transported, just as (50b) involves four thousand events of a ship crossing the lock.

(50) a. That airline transported 10 million passengers last year.
b. Four thousand ships passed through the lock last year.

The event-based account of (50) cannot be extended to (49), however. Although it is in principle possible to imagine that *three* is counting interviewing events rather than interviewees, the subject would still have to refer to interviewees, since *be the same person* doesn't compare events. Furthermore, since floated *all* requires its associated DP to be plural, even if we were counting events, we would still need *the three people we interviewed* to denote a plurality of concepts of interviewees.

Krifka (2009) is particularly relevant to us, as he explicitly argues for Theory B over Theory A. The motivation comes from the behavior of nouns such as *outfit* in counting configurations. For example, from three shirts and two pairs of pants, six outfits can be formed. Because these six outfits materially overlap and cannot all exist simultaneously at the same point in time, it appears that when we say *I brought six outfits* we are not counting concrete objects but intensional entities. Krifka therefore proposes that *outfit* denotes a predicate of individual concepts, and explicitly argues that DPs such as *two outfits* must range over objects of Theory B rather than those of Theory A.

The argument comes from data such as the following:

(51) Two outfits are similar.

The point is that the above sentence can be true even if we're talking about two outfits that materially overlap. Under Krifka's proposal, we could represent these outfits as the concepts u and v. The issue is that if *similar* composes with the intensional plural in (52a), we would never be able to tell whether they are similar or not, because the material overlap prevents this object to ever be defined. He thus concludes that *similar* composes with a plurality of concepts such as (52b).

(52) a. $\lambda i.\, \{u(i), v(i)\}$
b. $\{u, v\}$

Krifka's argument is thus complementary to ours: both reach the same conclusion, but via different routes — his through the semantics of *outfit* and similar nouns, ours through plural NPs within the subject of collective intensional VPs.[3]

4 Are intensional plurals necessary?

In the previous section, we presented three arguments in favor of a grammar that can generate pluralities of concepts, i.e., Theory B. However, given the type-flexible meaning we assigned to our plural operators, Theory B can also generate intensional plurals. The question we address now is whether one can get away with just pluralities of concepts. That is, perhaps the right conclusion is not that pluralities exist across two different domains, but rather that we should "generalize to the worst case" and assume that all pluralities are pluralities of concepts. In this section, we argue against this conclusion: intensional plurals are still needed. We furthermore show that deriving intensional plurals via the **flat** function from §2 overgenerates, and this motivates two independent compositional routes for creating these objects.

The case for intensional plurals begins with a detour. Consider the following scenario: on Monday, Ann's shelf contains the books ABCD, in this order; on Tuesday, her shelf contains the books BDEF, in this order. As observed in Nathan 2006, under these circumstances, (53) would be false — the intuition being that only two books on the shelf changed from Monday to Tuesday.

(53) Every book on Ann's shelf changed. *false*

This is surprising under our analysis of *book* as a set of individual concepts: the IC operator, repeated in (54), would yield the predicate of concepts in (55), which is true of concepts such as *the leftmost book on the shelf* and *the second leftmost book on the shelf*. If these concepts were allowed to be in the domain of quantification of *every book on the shelf*, then, (53) would be incorrectly predicted to be true.

(54) $[\![\text{IC}]\!]^t = \lambda N.\lambda u.\, \forall i \in \mathsf{dom}(u).\, N(i)(u(i))$

[3] We are not certain that ICs are the most suitable way to model outfits. Krifka proposes that the outfit made out of the shirt s and the pair of pants p would denote the concept (λi : $\{s, p\}$ are arranged into an outfit in i. $\{s, p\}$). However, if s is a shirt that can also be worn inside out, then there are in fact two distinct outfits, but they would denote the same IC under this analysis.

(55) $[\![\text{IC}]\!]^t([\![\text{^[book on Ann's shelf]}]\!]^t)$
$= \lambda u.\, \forall i \in \text{dom}(u).\, [\![\text{book on Ann's shelf}]\!]^i(u(i))$

The solution in Nathan 2006 is to adopt a type shifter other than IC that would yield a predicate of ICs that would not be true of these problematic concepts. We set these details aside, since this constraint on IC is independent of our main argument.

What is relevant to our current discussion is that, in contrast to (53), sentence (56) has a true interpretation when uttered in the very same context. The intuition is that, under this reading, (56) conveys that the set of books on Ann's shelf on Monday is not identical to the set of books on Ann's shelf on Tuesday.

(56) The books on Ann's shelf changed.

It thus seems that *change* has a collective reading which makes (56) true in our scenario. This is further confirmed by the fact that adding a floated *each* to (56) makes it false:

(57) The books on Ann's shelf each changed (on a different hour of the day).[4]

On this collective reading, *change* composes with an intensional plural: as shown below, it needs to check whether the plural individual denoted by *the books on Ann's shelf* is the same at two distinct points in time.

(58) $[\![\text{the books on Ann's shelf changed}]\!]^t$
$= \{x \mid \text{book.on.shelf}(t_{\text{beg}})(x)\} \neq \{x \mid \text{book.on.shelf}(t_{\text{end}})(x)\}$

We account for these readings by assigning *change* a type-flexible reading in (59), which composes with any intensional object and compares its value at two different points in time:

(59) $[\![\text{change}_{s\alpha}]\!]^t = \lambda a.\, a(t_{\text{beg}}) \neq a(t_{\text{end}})$ $\qquad (s \to \alpha) \to t$

The LF for (56) is (60): $\textit{change}_{s\{e\}}$ composes with the intension of *the books on the shelf*.

(60) $\text{^[the}_e \text{ books on the shelf]}\ \text{changed}_{s\{e\}}$

We have now established that intensional plurals are needed. The remaining question is how they should be compositionally derived — in particular, whether they require the compositional route in (60) or whether we could instead add the following operator to our grammar:

(61) $[\![\text{FLAT}]\!]^i = \lambda U.\lambda i.\, \{\, u(i) \mid u \in U \,\}$
$= \textbf{flat}$

This would have the advantage of simplifying our view of plurality: we could dispense with our type-flexible plural operators, and assume they only create pluralities of individual concepts. Collective interpretations of *change* would then be derived by first building a plurality of concepts and then flattening it:

[4] The adverbial PP is added to improve the acceptability of floated *each*.

	2025	2026
Maine	Ann	Beth
Vermont	Beth	Ann
Massachusetts	Cleo	Cleo
Connecticut	Deb	Deb

Table 3: Governors in 2025–2026

(62) [FLAT [the$_{\{se\}}$ Δ_{se} ^[book on the shelf]]] changed$_{s\{e\}}$

However, the expectation would be that any DP denoting a plurality of concepts could be flattened into an intensional plural. This, we will now show, is empirically problematic, and the way to show this is to construct cases in which availability of FLAT would make a missing reading available.

To that effect, consider Table 3, which shows the governors of four U.S. states in 2025 and 2026. Under this scenario, sentence (63) is true in its collective reading: the two people who are the governors of Maine and Vermont are the same in 2025 and 2026.

(63) The two governors of Maine and Vermont remained the same.

In contrast, (64) sounds completely contradictory: its only interpretation is that the two governor-concepts that changed didn't change.

(64) # The two governors that changed remained the same.

A possible explanation of the contrast between (63) and (64) concerns the internal composition of the subject noun phrase. The relative clause *that changed* must be interpreted as a predicate of pluralities of concepts, thus forcing *governors* to also be interpreted as such. The LF for the subject of (64) is thus as follows:

(65) the$_{\{et\}}$ [two [Δ_{se} IC ^governors] [that both changed]]

Under our scenario, this DP denotes the plurality of concepts composed of the ICs *the governor of Maine* and *the governor of Vermont*. If we could apply FLAT to (65), in this scenario (64) would be true for the same reason (63) is: the intensional plural in (66) is true of the predicate *remained the same*, because it yields the plurality containing Ann and Beth in both 2025 and 2026.

(66) $[\![\text{FLAT}]\!]^t([\![(65)]\!]^t) = \lambda i.\{ x \mid \text{governor}(i)(\text{maine})(x) \vee \text{governor}(i)(\text{vermont})(x)\}$

The fact that it is impossible to make (64) coherent, let alone true, must then be attributed to the fact that FLAT is not available as a strategy to derive intensional plurals from pluralities of concepts. We thus conclude that intensional plurals and pluralities of concepts are both needed and we must generate them through independent compositional routes.

5 Concluding remarks

Our goal was to compare two ways of integrating plurality and intensionality in a single grammar, through an investigation of the denotation of plural noun phrases. One possibility is that such phrases denote functions from indices to pluralities (i.e., intensional plurals), and the other is that they denote sets of individual concepts (i.e., pluralities of concepts). We concluded that we need a grammar that is capable of generating pluralities of concepts, and that therefore the notion of plurality must extend beyond the domain of individuals (Schmitt 2019). Furthermore, we showed that the grammar must still be able to generate intensional plurals independently, a result that might seem surprising given that intensional plurals are derivable from pluralities of concepts via **flat**.

The strength of our argument depends on two premises: that plural individuals are needed to account for collective predication, and that individual concepts are needed to account for Partee's paradox. We showed that, if we take these as premises, pluralities of concepts follow as a necessary consequence. This is therefore a conditional result: we leave open what conclusions our data would support under alternative analyses of these phenomena.

An important issue that goes beyond the scope of our paper is that, while we showed that Theory A is not expressive enough, Theory B might in fact be *too* expressive. Even if it's sometimes possible to "overcount" individuals, there are many cases in which it isn't. For instance, if I had two meetings today, both with Ann, I cannot utter (67), even though Theory B predicts a parse to be available in which the subject denotes a non-trivial plurality of concepts (e.g., {*the person I met first*, *the person I met last*}).

(67) The people I met today are really smart.

Some of these issues could be blocked by appealing to the Maxim of Manner (Grice 1975), if we believe that plural noun phrases are structurally more complex than singular ones: (67) can be ruled out for being contextually equivalent to *the person I met today is really smart*. Manner alone is insufficient, however. Suppose that I had 5 meetings today, three of them with Ann and the other two with different people who were not particularly clever. In this scenario, I can't utter (68) to convey that most of my meetings were with a nice person (namely, Ann).

(68) Most people I met today are really smart.

The issue here is that the truth of (68) *depends* on overcounting — no simpler contextually equivalent alternative exists. We leave a full account of these cases for future research, but we are hopeful that a combination of pragmatic and semantic constraints will prove sufficient. The central result of the paper stands independently of these open questions: pluralities of concepts are necessary, and any adequate grammar of plurality must generate them.

References

Bennett, Michael Ruisdael (1974). "Some Extensions of a Montague Fragment of English". PhD Thesis. Los Angeles: University of California.

Condoravdi, Cleo, Richard Crouch, and Martin van den Berg (2001). "Counting Concepts". In: *Proceedings of the Thirteenth Amsterdam Colloquium*. Ed. by Robert van Rooij and Martin Stokhof. Amsterdam: ILLC/Department of Philosophy, University of Amsterdam, pp. 67–72.

Frana, Ilaria (2017). *Concealed Questions*. Oxford University Press. ISBN: 9780199670925. URL: http://dx.doi.org/10.1093/acprof:oso/9780199670925.001.0001.

Grice, H. Paul (1975). "Logic and Conversation". In: *Syntax and Semantics, Vol. 3: Speech Acts*. Ed. by Peter Cole and Jerry L. Morgan. Academic Press, pp. 41–58.

Grosu, Alexander and Manfred Krifka (2007). "*The Gifted Mathematician that You Claim to Be*: Equational Intensional 'Reconstruction' Relatives". In: *Linguistics and Philosophy* 30, pp. 445–485. URL: https://doi.org/10.1007/s10988-008-9022-5.

Gupta, Anil (1980). *The Logic of Common Nouns*. New Haven: Yale University Press.

Haslinger, Nina and Viola Schmitt (2021). "Distinguishing belief objects". In: *Formal approaches to number in Slavic and beyond*. Ed. by Mojmír Dočekal and Marcin Wągiel. Berlin: Language Science Press, pp. 251–274.

Keshet, Ezra (2010). "Split Intensionality: A New Scope Theory of de Re and de Dicto". In: *Linguistics and Philosophy* 33.4, pp. 251–283.

Krifka, Manfred (1990). "Four Thousand Ships Passed Through the Lock: Object-Induced Measure Functions on Events". In: *Linguistics and Philosophy* 13, pp. 487–520.

— (2009). "Counting Configurations". In: *Proceedings of Sinn und Bedeutung 13*. Ed. by Arndt Riester and Torgrim Solstad, pp. 309–324.

Link, Godehard (1983). "The logical analysis of plurals and mass terms: a lattice-theoretical approach". In: *Meaning, use and interpretation of language*. Ed. by Rainer Bäuerle, Christoph Schwarze, and Arnim von Stechow. Berlin: Moutn de Gruyter, pp. 302–323.

Montague, Richard (1973). "The Proper Treatment of Quantification in Ordinary English". In: *Approaches to Natural Language: Proceedings of the 1970 Stanford Workshop on Grammar and Semantics*. Ed. by K. J. J. Hintikka, J. M. E. Moravcsik, and P. Suppes. Dordrecht: Springer Netherlands, pp. 221–242. ISBN: 978-94-010-2506-5. URL: https://doi.org/10.1007/978-94-010-2506-5_10.

Nathan, Lance Edward (2006). "On the interpretation of concealed questions". PhD thesis. Massachusetts Institute of Technology.

Partee, Barbara H (1986). "Noun phrase interpretation and type-shifting principle". In: *Studies in Discourse Representation Theory and the Theory of Generalized Quantifiers*. Ed. by Jeroen Groenendijk, D. de Jongh, and Martin Stokhof. Dordrecht: Foris, pp. 115–143.

Percus, Orin (2019). "Uli and our generation: some reminiscences". In: *Snippets* 37. Ed. by Andreea C. Nicolae, Patrick D. Elliott, and Yasutada Sudo, pp. 82–83.

Schmitt, Viola (Nov. 2019). “Pluralities across categories and plural projection”. In: *Semantics and Pragmatics* 12.17, pp. 1–55.

Schwager, Magdalena (2007). “Bodyguards under Cover: The Status of Individual Concepts”. In: *Proceedings of Semantics and Linguistic Theory 17*. Ed. by M. Gibson and T. Friedman. Cornell University. Ithaca, NY.

Winter, Yoad (2001). *Flexibility Principles in Boolean Semantics: The Interpretation of Coordination, Plurality, and Scope in Natural Language*. Cambridge, Massachusetts: MIT Press.

Zhang, Linmin (2016). “External and internal *same*: a uniform analysis motivated by attitude reports”. In: *Proceedings of Sinn und Bedeutung 20*. Ed. by Nadine Bade, Polina Berezovskaya, and Anthea Schöller, pp. 833–850. URL: https://doi.org/10.18148/sub/2016.v20i0.298.